

In 2007, K. Győry and K. Yu proved a very important theorem on S -unit equations in number fields. As an application, Győry [Gy] derived the following results concerning the abc conjecture for a number field \mathbb{K} . Let $\log^* x = \max(\log x, 1)$ if $x > 0$.

Theorem 1 *Let $a, b, c \in \mathbb{K}^*$ such that $a + b + c = 0$. Then*

$$\log H_{\mathbb{K}}(a, b, c) < c_1 |D_{\mathbb{K}/\mathbb{Q}}|^{3/2} (\log^* |D_{\mathbb{K}/\mathbb{Q}}|)^{3[\mathbb{K}:\mathbb{Q}]-1} \frac{P}{\log^* P} N^{[\mathbb{K}:\mathbb{Q}]\alpha}$$

where

$$\alpha = (c_2 \log^* |D_{\mathbb{K}/\mathbb{Q}}| + 19.2 \log(\log(\log N^*))) / \log(\log N^*), \quad N = \text{rad}_{\mathbb{K}/\mathbb{Q}}(a, b, c), \quad N^* = \max(N, 16),$$

and P is the greatest factor $N(\mathfrak{p})$ where $\mathfrak{p} \in I_{\mathbb{K}}(a, b, c)$, and c_1 and c_2 are explicit positive constants depending only on $[\mathbb{K} : \mathbb{Q}]$.

Moreover, Győry [Gy] provides the following nice result in the direction of the uniform abc conjecture for number fields.

Theorem 2 *Let $a, b, c \in \mathbb{K}^*$ such that $a + b + c = 0$ and $N = \text{rad}_{\mathbb{K}/\mathbb{Q}}(a, b, c)$. For every $\varepsilon > 0$, there is a positive constant $c_3 = c_3(\mathbb{K}, \varepsilon)$, which depends only on \mathbb{K} and ε such that*

$$\log H_{\mathbb{K}}(a, b, c) < c_3(\mathbb{K}, \varepsilon) N^{1+\varepsilon}.$$

Further, if

$$N > \max(\exp \exp(\max(|D_{\mathbb{K}/\mathbb{Q}}|, e)), |D_{\mathbb{K}/\mathbb{Q}}|^{2/\varepsilon}),$$

then

$$\log H_{\mathbb{K}}(a, b, c) < c_4(|D_{\mathbb{K}/\mathbb{Q}}|N)^{1+\varepsilon}$$

with an effectively computable positive constant $c_4 = c_4([\mathbb{K} : \mathbb{Q}], \varepsilon)$ depending only on $[\mathbb{K} : \mathbb{Q}]$ and ε .