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Essouabri, D. (F-NANC-IE)

Prolongement méromorphe des séries de Dirichlet à support dans un sous-ensemble semi-algébrique de R^n. (French. English summary) [Meromorphic continuation of Dirichlet series with support in a semialgebraic subset of R^n]

Compositio Math. **114** (1998), *no. 3,* 219–261.
11M41 (11P21)

Let A be an open semi-algebraic subset of $\mathbf{R}^k$ (i.e., defined by polynomial inequalities), P a polynomial in k variables with real coefficients, and s a complex variable. Continuing his earlier work [Ann. Inst. Fourier (Grenoble) **47** (1997), no. 2, 429–483; MR 99d:11098], the author considers generalized Dirichlet series

$$Z(P,A;s)=\sum_{\mathbf{m}\in A\cap\mathbf{Z}^k}P(\mathbf{m})^{-s}\,.$$

He is interested in their domain of convergence, the possibility of a meromorphic continuation, and in estimates for the growth of $s\mapsto Z(P,A;s)$ in vertical strips outside the domain of convergence. The results obtained are deep, fairly general, and they admit applications to the theory of lattice points in large regions. These are obtained in the form

$$\sum_{\substack{\mathbf{m}\in A\cap\mathbf{Z}^k\\ P(\mathbf{m})\le t}}1=t^{\sigma_0}Q(\log t)(1+O(t^{-\omega}))\,,$$

where t is a large real variable, $\sigma_0=\sigma_0(P,A)$ is the abscissa of convergence of $Z(P,A;s)$, Q is a certain polynomial and ω some positive constant.

The essence of this very interesting paper (whose details are too involved to be be stated in full in this review) is that certain classical results from lattice point theory in the sense of Landau (in a crude form) remain true under very mild and general hypotheses.

Reviewed by *Werner Georg Nowak*

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