French-japanese Workshop on zeta functions

Titles ans abstracts of the talks:

- **Michel Balazard (Bordeaux, France):**

  "Une utilisation de dzêta, fonction de variable réelle"

  Abstract: "Je présente une généralisation de l'identité de Selberg (dans la théorie élémentaire des nombres premiers) à l'aide de polynômes d'Appell et de la fonction dzêta considérée pour les valeurs réelles de la variable."

- **Gautami Bhowmik (Lille, France):**

  "Natural boundaries of Dirichlet's series."

  Abstract: TBA.

- **Régis de la Bretèche (Paris VII, France):**

  "Fonctions zêta des hauteurs et points rationnels sur les variétés algébriques."

  Abstract: "We can associate a Dirichlet series to the number of rational points of an algebraic variety V. We will expose some recent results on their analytic properties obtained when V is a certain cubic surface (in collaboration with Swinnerton-Dyer and also with Browning-Derenthal)."

  (This talk will be in French)

- **Stéphane Fischler (Paris XI, France):**

  "Irrationality results about multiple zeta values."

  Abstract: "In this lecture, I will give a survey of the known results of irrationality, or linear independence over the rationals, of multiple zeta values. At the end, I will also state some new results of non-diophantine nature (obtained with J. Cresson and T. Rivoal) that could be useful for obtaining new irrationality results."

- **Ken Kamano (Waseda, Japan):**

  "A q-analogue of the von Staudt-Clausen theorem."

  Abstract: "In this talk, we introduce the q-Bernoulli numbers on C_p, the completion of the algebraic closure of the field of p-adic numbers, and give a q-analogue of the von Staudt-Clausen theorem by using p-adic integral. When q tends to 1, we obtain the original von Staudt-Clausen theorem."

- **Stéphane Louboutin (Marseille, France):**

"Simple proofs of the Siegel-Tatuzawa and Brauer-Siegel theorems."

Abstract: "We give a simple proof of the Siegel-Tatuzawa theorem according to which the residues at $s = 1$ of the Dedekind zeta functions of quadratic number fields are effectively not too small, with at most one exceptional quadratic field. We then give a simple proof of the Brauer-Siegel theorem for normal number fields which gives an asymptotic for the logarithm of the product of the class number by the regulator of number fields."

- **Kohji Matsumoto (Nagoya, Japan):**

"Recursive relations among zeta-functions of root systems and Dynkin diagrams."

Abstract: "The Witten zeta-function, associated with any semisimple Lie algebra, was first introduced in connection with a problem in quantum gauge theory. It can be written as an r-ple zeta-function of a single variable, where r is the rank of the underlying algebra. However, in order to study the analytic properties, it is more flexible to introduce the multi-variable zeta-functions associated with root systems, or more general sets of roots. In this talk I will report that the family of those zeta-functions has a recursive structure described by the Mellin-Barnes integral formula, and that the structure can be explained in terms of Dynkin diagrams."

- **Takashi Nakamura (Nagoya, Japan):**

"Relations for double L-values and functonal relations for Witten zeta functions."

Abstract: "In this talk, we show functional relations for Witten zeta functions by using a double L-value relation. By these functional relations, we obtain new proofs of known result on the Tornheim double zeta function and the Euler- Zagier double zeta function and their alternating and character analogues, and also the Euler-Zagier triple zeta function."

- **Emmanuel Peyre (Grenoble, France):**

"Fonctions zêtas des hauteurs des variétés de drapeaux."

Abstract: "Les fonctions zêtas des hauteurs permettent d'étudier le comportement asymptotique du nombre de points de hauteur bornée sur les variétés projectives. Elles sont définies par la série

$$\zeta_{V,H}(s) = \sum_{x \in V(K)} \frac{1}{H(x)^s}$$

où $V(K)$ désigne l'ensemble des points rationnels de V et $H : V(K) \to \mathbf{R}$ est une hauteur sur V.

Dans le cas des variétés projectives homogènes sous un groupe linéaire, lorsque la hauteur est choisie de façon convenable, la fonction zêta des hauteurs coïncide avec une série d'Eisenstein, si bien que ses propriétés sont particulièrement bien connues. Cette coïncidence permit à Franke, Manin et Tschinkel d'élucider le comportement asymptotique des points de hauteur bornée sur ces variétés.

L'objet de cet exposé est de présenter cette approche aussi bien dans le cas arithmétique que dans le cas fonctionnel."

- **Tanguy Rivoal (Grenoble, France):**

"Interpolation lagrangienne et fonction zêta de Riemann."

Abstract: "Gel'fond a montré la transcendance de $\exp(\pi)$ en développant la fonction exponentielle en une série d'interpolation polynomiale aux point de $\pi\mathbb{Z}[i]$. J'expliquerai comment adapter ce type de preuve pour montrer l'irrationalité de $\log(2)$, $\zeta(2)$ et $\zeta(3)$ au moyen de séries d'interpolation rationnelle liées à la fonction zêta d'Hurwitz.

Gel'fond proved the transcendence of $\exp(\pi)$ by mean of the expansion of the exponential function in polynomial interpolation series at the points of $\pi\mathbb{Z}[i]$. I will explain how to adapt this kind of proof to show the irrationality of $\log(2)$, $\zeta(2)$ and $\zeta(3)$ by mean of rational interpolation series related to Hurwitz zeta function."

- **Hirofumi Tsumura (Tokyo, Japan):**

"A method of producing functional relations for various multiple zeta-functions."

Abstract: "In this talk, we introduce some methods of producing functional relations for various types of multiple zeta-functions. Considering special values, we give new evaluation formulas for these multiple zeta values. Most parts of this talk are the joint work with Kohji Matsumoto and Yasushi Komori."

- **Brigitte Vallée (Caen, France):**

"Dynamical Zeta functions arising in the analyses of Euclid Algorithms: existence of a strip without poles and its consequences for the analyses."

Abstract: "The Euclid Algorithm is, according to Knuth, the grandfather of all the algorithms. It is of great use, and the basic "parameters" of the algorithm, namely the number of iterations, the evolution of remainders, or the total number of elementary operations, are of great interest, since they describe the precise complexity of the algorithm.

However, there exist very natural questions about the probabilistic behaviour of the algorithm which had not received a precise answer until recently.

All the recent results are obtained by using the same methodology. First, as usual, Dirichlet generating functions $T(s)$ of the main parameters are considered. Then, provided that the position and the nature of their singularities are known, it is possible to extract coefficients from these Dirichlet series: This allows to obtain the probabilistic behaviour of the main parameters. Now, the main question is "How to obtain this knowledge about singularities?" The answer is provided by the underlying dynamical system, and, more notably, by the transfer operators $\mathbf{H}_s$ of the system, since it is possible to obtain an alternative expression of the main generating Dirichet series as a function of the quasi-inverse $(I-\mathbf{H}_s)^{-1}$ of the transfer operators (plain transfer operators or modified ones). Then, there is a close connection between the (position and the nature of the) singularities of the Dirichlet series and the main spectral properties of the transfer operator.

There are now two kinds of probabilistic questions about the main parameters of the algorithms. If we are only interested in the "average-case analysis", then Tauberian Theorems are used for extracting coefficients, and an only elementary knowledge about spectral properties of the transfer operators is needed. In particular, we do not need studying the quasi-inverse $(I - \mathbf{H}_s)^{-1}$ at the left of the vertical line $\Re s = 1$. On the other hand, if we are interested in distributional analyses, then the Perron Formula is used, and a sharpest knowledge about spectral properties of the transfer operator is needed. The existence of free of pôles strip at the left of the line $\Re s = 1$ plays a crucial rôle in the distributional analyses.

Recently, for a general dynamical system, Dolgopyat relates the existence of such a strip for the quasi-inverse $(I - \mathbf{H}_s)^{-1}$ to deep properties of the dynamical itself, namely the so-called *UNI* Property which expresses that the branches of the dynamical systems are not too close to affine branches. Baladi and Vallée (03) extended the result of Dolgopyat to the case of a dynamical system with an infinite number of branches, and also obtain a generalization for variants of the quasi-inverse.

All these results lead to a very precise knowledge of the distributional behaviour of the main parameters of the Euclid Algorithm."