Chevalley groups, reflections groups, braid groups
January 24th-28th 2011

Program

Monday
08h30-09h30  G. Lusztig: On the cleanness of cuspidal character sheaves.
09h45-10h45  P. Achar: Derived Satake equivalence and geometric restriction to a Levi subgroup.
11h15-12h15  G. Williamson: Generators and relations for Soergel bimodules.
16h30-17h30  C. Bonnafé: Calogero-Moser cells.
17h45-18h45  M. Broué: News from Spetses.

Tuesday
08h30-09h30  O. Brunat: On semisimple classes in finite reductive groups.
09h45-10h45  R. Charney: Groups Associated to Random Graphs.
11h15-12h15  M. Chlouveraki: Canonical basic sets and cell modules for Hecke algebras.
17h00-18h00  L. Paris: Basic questions on Artin-Tits groups.
18h15-19h15  M. Geck: On the Kazhdan–Lusztig ordering of characters and cells.

Wednesday
08h30-09h30  D. Juteau: Latest news about parity sheaves.
09h45-10h45  B. Leclerc: Quantum affine algebras and cluster algebras.
11h15-12h15  G. Lehrer: The second fundamental theorem of invariant theory.

Thursday
08h30-09h30  G. Malle: Decomposition of $R^G_L$ and quasi-isolated blocks.
09h45-10h45  I. Marin: Galois automorphisms of complex reflection groups.
11h15-12h15  P. Dehornoy: A conjecture about Artin-Tits groups.
17h00-18h00  M. Picantin: Garside structures for braid groups of complex reflection groups.
18h15-19h15  R. Rouquier: Generic representations and perverse equivalences.

Friday
08h30-09h30  T. Shoji: Character sheaves associated to the enhanced nilpotent cone.
09h45-10h45  D. Bessis: Chasing the Rabbit.
11h15-12h15  P. Cartier: Des groupes de tresses au groupe de Grothendieck-Teichmüller.
Mise au point sur les résultats récents.
Titles and Abstracts of the talks

Pramod Achar: Derived Satake equivalence and geometric restriction to a Levi subgroup.

Let $G$ be a complex reductive algebraic group. Let $N$ denote the nilpotent cone in its Lie algebra, and let $Gr$ denote the affine Grassmannian of its Langlands dual group. The celebrated geometric Satake equivalence is an equivalence of tensor categories between spherical perverse sheaves on $Gr$ and representations of $G$. Following methods of Arkhipov-Bezrukavnikov-Ginzburg, this can be extended to an equivalence of triangulated categories between the spherical derived category of perverse sheaves on $Gr$ and the perfect derived category of coherent sheaves on $N$. It is natural to ask, ”Is this equivalence compatible with restriction to a Levi subgroup?” There are surprising subtleties involved in even making this question precise, essentially because the spherical derived category on $Gr$ is the ”wrong” category from the viewpoint of the Weil conjectures. I will explain these subtleties and how one may overcome them, leading to a positive answer to the question above. This is joint work with S. Riche.

David Bessis: Chasing the Rabbit.

There are 34 exceptional complex reflection groups, and some are more exceptional than others. The nastiest exception in the barn may very well be $G_{31}$, aka the Rabbit, a four-dimensional group that needs five generating reflections. It is nicknamed after a conjectural presentation for its braid group $B_{31}$, symbolized by a diagram with a round face and two big ears. Jean and I spent nearly two years writing a software that could perform exact monodromy computations and formally prove this conjecture. A much more precise picture of the Rabbit’s anatomy can be achieved by viewing $B_{31}$ within the braid group of type $E_8$. The complete dissection involves interesting paraphernalia (Garside nerves, cyclic homology, cycling sieving phenomenon, Kerekjarto-Springer theory in braid categories), which I will try to present.

Cédric Bonnafé: Calogero-Moser cells.

Joint work with Raphael Rouquier : we propose a construction of a partition of a finite complex reflection group $W$ into Calogero-Moser cells, built on the representation theory of rational Cherednik algebras at $t=0$. Whenever $W$ is a Coxeter group, we provide some numerical and theoretical evidences that the Calogero-Moser cells should look like (and maybe be equal) to Kazhdan-Lusztig two-sided cells.
Michel Broué: News from Spetses.

We shall lecture on part of the current situation concerning the Spets Program. In particular, we shall introduce the twin notions of “compact type spets” and “noncompact type spets” (those twins are identical for real spetses). We shall also state and describe an existence and unicity theorem for spetses associated with exceptional complex reflection groups.

Olivier Brunat: On semisimple classes in finite reductive groups.

This talk is concerned with the semisimple classes of finite reductive groups. Let $G$ be a connected reductive group and let $F$ be a Frobenius map on $G$. When the derived subgroup of $G$ is simply-connected, Steinberg has computed the number of semisimple classes of the finite group $G^F$. We will extend this result to any finite reductive groups. We will also compute the number of semisimple classes of $G^F$ whose representatives have a disconnected centralizer in $G$. Our main tools are the theory of Gelfand-Graev characters for groups with disconnected center developed by Digne-Lehrer-Michel and the geometry of the Brauer complex.


On doit à Ihara la définition d’une algèbre de Lie liée à la tour des groupes de tresses. Il est connu que cette algèbre de Lie est reliée aux associateurs de Drinfeld, et au groupe de Grothendieck-Teichmüller. Je voudrais revenir sur les résultats récents de FURUSHO sur les associateurs et de Francis BROWN sur les motifs de Tate mixtes, et faire le panorama des divers groupes pro-unipotents connus. Ceci permettra de formuler un programme de recherche réaliste, et de préciser les enjeux pour le proche avenir.

Ruth Charney: Groups Associated to Random Graphs.

To a finite graph $\Gamma$, one can associate a right-angled Coxeter group or a right-angled Artin group whose generators are the vertices of $\Gamma$ with commutator relations between adjacent vertices. More generally, assigning arbitrary finitely generated groups to the vertices of $\Gamma$, one defines a graph product of groups similarly. In joint work with Michael Farber, we consider graph products of groups associated to random graphs. Using the model of a random graph as a graph with $n$ vertices, any two of which span an edge with specified probability $p_n$, we investigate properties of these groups as $n$ approaches infinity.
**Maria Chlouveraki:** *Canonical basic sets and cell modules for Hecke algebras.*

In this talk, we will try to construct canonical basic sets and cell modules for Hecke algebras through category O for the rational Cherednik algebra and the KZ-functor.

**Patrick Dehornoy:** *A conjecture about Artin-Tits groups.*

We conjecture that the word problem of Artin-Tits groups can be solved without introducing trivial patterns $ss^{-1}$ or $s^{-1}s$. We shall make this statement (which does not imply the decidability of the word problem) precise, and discuss particular cases and consequences.

**Meinolf Geck:** *On the Kazhdan–Lusztig ordering of characters and cells.*

Let $W$ be a finite Coxeter group. The Kazhdan-Lusztig theory of cells gives rise to a partition of $\text{Irr}(W)$ into families and to a natural partial ordering on the families. We show that the partition and the partial order are determined purely in terms of the character tables of $W$ and its parabolic subgroups.

**Daniel Juteau:** *Latest news about parity sheaves.*

In joint work with Carl Mautner and Geordie Williamson, we introduced parity sheaves as a replacement for intersection complexes when working with coefficients in positive characteristic, with a view towards applications in representation theory. We had proved that, under certain parity conditions which are typically satisfied in representation theory, they are classified in the same way as the intersection complexes. Recently, we realized that we could give a more general definition involving a parity function depending on the strata, and that one can use a construction “la Deligne” to prove that they always exist. In particular, the new definition includes the tilting sheaves of Beilinson, Bezrukavnikov and Finkelberg.

**Bernard Leclerc:** *Quantum affine algebras and cluster algebras.*

This talk will give an overview of some conjectural description of the Grothendieck ring of the category of finite-dimensional representations of a quantum affine algebra, in terms of cluster algebras. This conjecture generalizes classical results of Chari and Pressley for $\hat{sl}_2$. It is supported by some partial results of Hernandez-Leclerc, and of Nakajima.

**Gus Lehrer:** *The second fundamental theorem of invariant theory.*

I shall explain how the commutant of the classical orthogonal group action on tensor space has a presentation which is obtained from the Brauer algebra by adding one single idempotent relation.
George Lusztig: *On the cleanness of cuspidal character sheaves.*

Gunter Malle: *Decomposition of $R^G_r$ and quasi-isolated blocks.*

Ivan Marin: *Galois automorphisms of complex reflection groups.*

In a joint work with Jean Michel a few years ago, we algebraically described the automorphism group of a complex (pseudo-)reflection group $W$. It contains many automorphism connected to the Galois group of the field of definition of $W$, which thus deserve the name of Galois automorphisms. One of the applications we got is that the invariants of $W$ are defined over the field $Q$ of rational numbers. In this talk I will first describe further applications of our work, and then I will present these Galois automorphisms in a more geometric context.

Luis Paris: *Basic questions on Artin-Tits groups.*

Artin-Tits groups are the groups that admit presentations with relations of the form $aba \cdots = bab \cdots$, the terms on the left hand side and on the right and side having the same length. These were introduced by Tits in the 60s as extensions of Coxeter groups, but it is in the 70s with seminal works by Brieskorn, Saito, Deligne and others that they acquired their importance. The theory of Artin-Tits groups is quite incomplete in the sense that there is no known result valid for all these groups, and the theory consists on the study of some more or less extended families.

We will give an overview on some basic questions on Artin-Tits groups such as the word problem, the center, and the torsion, and on a more specific question to the theory: the $K(\pi,1)$ problem.

Matthieu Picantin: *Garside structures for braid groups of complex reflection groups.*

In this talk, we will investigate those properties shared by the posets defined by multiple reflection lengths in complex reflection groups in order to provide Garside structures for the associated braid groups.

Raphael Rouquier: *Generic representations and perverse equivalences.*

We will explain how perverse equivalences can be used as a substitute for certain Deligne-Lusztig varieties. The generic properties of the cohomology of those varieties translate into properties of modular representations of tori and relative reflection groups. This is joint work with David Craven.
Toshiaki Shoji: Character sheaves associated to the enhanced nilpotent cone.

Let $V$ be an $n$ dimensional vector space over a finite field $\mathbb{F}_q$, and $G = GL(V)$ with Frobenius map $F$ on $G$ and on $V$. We construct certain simple perverse sheaves on $G \times V^{r-1}$, which are $G$-equivariant with respect to the diagonal action of $G$, and call them character sheaves associated to the enhanced nilpotent cone $\mathcal{N} \times V^{r-1}$, where $\mathcal{N}$ is the usual nilpotent cone on $V$. Then we define Green functions by making use of those character sheaves, which are $G^F$-invariant functions on $(\mathcal{N} \times V^{r-1})^F$. We show that such Green functions are closely related to the Kostka functions associated to the complex reflection group $G(r, 1, n)$.

Geordie Williamson: Generators and relations for Soergel bimodules.

I will discuss work in progress with Ben Elias in which we present the monoidal category of Soergel bimodules via generators and relations (building on earlier work of Libedinsky and Elias-Khovanov). I will explain why an important role in the story is played by actions of Coxeter groups on categories.