On fields of algebraic numbers with bounded local degrees.

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Abstract:

We provide a characterization of infinite algebraic Galois extensions of the rationals with uniformly bounded local degrees.

In particular we show that for an infinite Galois extension of the rationals the following three properties are equivalent: having uniformly bounded local degrees at every prime; having uniformly bounded local degrees at almost every prime; having Galois group of finite exponent. The proof of this result enlightens interesting connections with Zelmanov’s work on the Restricted Burnside Problem. We give a formula to explicitly compute bounds for the local degrees of an infinite extension in some special cases.

We relate the uniform boundedness of the local degrees to other properties: being a subfield of $\mathbb{Q}^{(d)}$, which is defined as the compositum of all number fields of degree at most $d$ over $\mathbb{Q}$; being generated by elements of bounded degree. We prove that the above properties are equivalent for abelian extensions, but not in general; we provide counterexamples based on group-theoretical constructions with extraspecial groups and their modules.