17th International Conference on Fibonacci Numbers and Their Applications

Université de Caen Normandie
Caen, France
June 27–July 2, 2016
Revised on Wednesday 22nd June, 2016 at 19:17

Contents

Monday morning registration and all talks, regular, plenary and open-to-the-public, are scheduled to take place at the **Amphi 057** of the **S3** building.

Speakers whose name is stamped with a double asterisk are Bruckman eligible.

Monday, June 27

Morning Session moderated by **Bill Webb**

8:30 AM **Conference registration in Amphi 057 of the S3 building**

9:30 AM Welcoming words from the University **Vice-President** Anne Guesdon

10:00 AM **Peter G. Anderson:** *Notes & Extensions for a Remarkable Continued Function*

10:20 AM **Curtis Cooper:** *On Identities of Ruggles, Horadam, and Howard*

10:40 AM **Paul K. Stockmeyer:** *An Exploration of Sequence A000975*

11:00 AM **Paul Young:** *Congruences for Bernoulli-Lucas sums*

11:20 AM **Break**

11:30 AM **Maciej Gawron**:** *Arithmetic properties of the convolutions of the Prouhet-Thue-Morse sequence*

11:50 AM **Piotr Miska**:** *Arithmetic properties of convolutions of binary partition function*

12:10 PM **Mustafa Asci:** *On Gaussian Fibonacci and Gaussian Lucas numbers*

12:30 PM **Lunch**

Monday, June 27

Afternoon Session moderated by **Andreas Philippou** and **Paul Young**

2:00 PM **Takao Komatsu:** *Reciprocal sums of sequences involving balancing and Lucas-balancing numbers*

2:20 PM **Bijan Patel:** *The Period Modulo Product of Consecutive Balancing Numbers*

2:40 PM **Gopal K. Panda:** *Sequences associated with balancing-like sequences*

3:00 PM **Spyros Dafnis and Andreas Philippou:** *Infinite sums of weighted Fibonacci numbers of order k*

3:20 PM **Break**

3:40 PM **Levent Kargin:** *Convolution identities for Fubini polynomials*

4:00 PM **Claudio Pita Ruiz:** *Explicit Formulas for Weighted Sums of Squares via Generalized Eulerian Polynomials*

4:20 PM **Mehmet Cenkci:** *Convolution Identities for Degenerate Bernoulli Polynomials*

4:40 PM **Marc Chamberland:** *Combinatorial Identities via Matrix Factorization*

7:30 PM–10:00 PM **Wine & Cheese Reception at the City Hall**

Tuesday, June 28

Morning Session moderated by **Peter Anderson**

10:00 AM **William Webb:** *Some Surprising Lacunary Binomial Sums*

10:20 AM **Tamas Lengyel:** *On the rate of p-adic convergence of alternating sums of powers of binomial coefficients*

10:40 AM **Jonathan Chappelon******:** *Balanced binary triangles generated from periodic sequences*

11:00 AM **Zvonko Cerin:** *On Candido Identity*

11:20 AM **Break**

11:45 AM **Miho Aoki:** *Mod p equivalence classes of linear recurrences of degree two*

12:05 PM **Pantelimon Stănică:** *A new approach to the Golay-Rudin-Shapiro sequence and variations*

12:30 PM **Lunch**

Tuesday, June 28

Afternoon Session moderated by **Christian Ballot**

2:00 PM **Anitha Srinivasan:** *On Markoff numbers*

2:20 PM **Nathan Hamlin:** *The Combinatorial Nature of Numerical Representation*

2:40 PM **Paul Cull:** *What I tell you K times is True …*

3:00 PM **Steven J. Miller, Zhao Pan**, Huanzhong Xu**:** *Convergence Rates in Generalized Zeckendorf Decompositions*

3:30 PM **Problem Session** by Clark Kimberling

4:30 PM **Teurgoule Break**

General Public Session

This session is open to the public

5:00 PM **Andreas Hinz:** *De Hanoï à Londres, aller et retour*

5:25 PM **Ron Knott:** *Numbers You Can Eat: nature's numbers that you see and eat in your everyday fruit and vegetable*

5:50 PM **Arthur Benjamin:** *MatheMagics*

Wednesday, June 29

Morning Session moderated by **Vincent Bosser**

9:00 AM **Édouard Lucas' Memorial Lecture**

Jean-Paul Allouche: *Variations on the Fibonacci Binary Sequence*

10:00 AM **Break**

10:20 AM **Gabriella Rácz** and Gábor Nyul**:** *Combinatorial and divisibility properties of generalized Lah numbers and their sums*

10:40 AM **Gábor Nyul** and Eszter Gyimesi:** *Recent results on generalizations of Bell numbers*

11:00 AM **Arthur T. Benjamin and Joel Ornstein**:** *A Bijective Proof of a Derangement Recurrence*

11:20 AM **Group Photo**
Meet in front of Amphi 057

11:40 AM–12:40 PM **Lunch**

Afternoon Activities

2:00 PM–4:00 PM **Downtown Caen or Abbaye-aux-Hommes's visits:**
Groups 1 & 2 (Downtown Caen visits)
Group 3 (Abbaye aux Hommes)

4:15 PM–5:15 PM **Degustation at the Palais Ducal (behind the Abbaye aux Hommes)**

Thursday, June 30

Morning Session moderated by **Takao Komatsu** and **Florian Luca**

8:40 AM	**Augustine O. Munagi:** *Two applications of the bijection on Fibonacci set partitions*
9:00 AM	**Barry Balof and Helen Jenne:** *Tilings, Continued Fractions, Derangements, Scramblings, and e*
9:20 AM	**Imène Benrabia:(?)** *Combinatorial interpretation of binomial coefficient analogues related to 3-Fibonacci sequence*
9:40 AM	**Dale Gerdemann:** *Two Combinatorial Interpretations for the Fibonomial*
10:00 AM	**Murat Sahin**:** *Fibonacci-Like Sequences and Chebysev Polynomials*

10:20 AM	**Break**

10:40 AM	**Mohammed Taous:** *On the* 2*-class group of* $\mathbb{Q}(\sqrt{5pF_p})$ *where* F_p *is a prime Fibonacci number*
11:00 AM	**Ö. Öztunç Kaymak**:** *A New Bound for the zeros of R-Bonacci Polynomials*
11:20 AM	**Tomislav Pejković:** *P-adic root separation for quadratic and cubic polynomials*
11:40 AM	**Chatchawan Panraksa:** *Boundedness of Periodic Points of Rational Functions with Good Reduction Everywhere by Using Fibonacci Numbers*
12:00 PM	**Keith Johnson:** *Rational polynomials that take integer values on the Fibonacci numbers* *+ 5-10 mns Halifax and Dalhousie U. presentation by Keith Johnson*

12:30 PM–2:00 PM **Lunch + Fibonacci Association Board Meeting**

Thursday, June 30

Afternoon Session moderated by **Heiko Harborth** and **Pante Stănică**

2:00 PM	**Márton Szikszai:** *A variant of the Brocard-Ramanujan equation for Lucas sequences*
2:20 PM	**Hidetaka Kitayama** and Daisuke Shiomi**:** *Perfect powers in Fibonacci and Lucas polynomials in finite fields*
2:40 PM	**Aram Tangboonduangjit**:** *Exact Divisibility Properties of Some Subsequences of the Mersenne Numbers*
3:00 PM	**Break**
3:30 PM	**Florian Luca:** *Diophantine triples of Fibonacci numbers*
3:50 PM	**Ayhan Dil and Mirac Cetin Firengiz:** *Shifted Euler-Seidel Matrices*
4:10 PM	**Thotsaporn Thanatipanonda:** *Generalized Fibonacci Numbers with Matrix Method*
4:30 PM	**Tamás Szakács**:** *k-order linear recursive sequences and the Golden Ratio*
7:00 PM	**Conference Banquet at the Pont de Coudray Auberge** Board buses Place Saint-Pierre

Friday, July 1

Morning Session moderated by Curtis Cooper

9:00 AM	**Clark Kimberling and Peter Moses:** *Polynomial Trees and Subtrees*
9:20 AM	**Andreas M. Hinz:** *The Lichtenberg Sequence*
9:40 AM	**Russell Hendel:** *Generalization of the Tagiuri-Gould Identities to m parameters with proof by Binetization*
10:00 AM	**Virginia Johnson and Charles Cook:** *Areas of triangles, quadrilaterals and other polygons with vertices from various sequences*
10:20 AM	**Charles Cook and Michael Bacon:** *Higher Order Boustrophedon Transforms for Certain Well–Known Sequences*
10:40 AM	**Break**
11:10 AM	**Bruce Boman:** *Why do Fibonacci numbers appear in patterns of growth in nature? Clues from modeling asymmetric cell division*
11:30 AM	**Burghard Herrmann:** *Phyllotactically Perfect Angles*
11:50 AM	**Christopher Brown:** *Phi and its Natural Logarithm*
12:10 PM	**Frédéric Mansuy:** *Construction of a Quasicrystalline Fivefold Structure*
12:30 PM	**Lunch**

Friday, July 1

Afternoon Session moderated by **Clark Kimberling**

2:00 PM **Bruckman Prizes Awards**

2:20 PM **Larry Ericksen:** *Properties of Fibbinary numbers with applications in Stern polynomials.*

2:40 PM **A. Shannon, C. Cook, R. Hillman:** *Basic linear recursive matrices*

3:00 PM **Rita Giuliano:** *Convergence results for the iterated means of the denominators of Lüroth expansions*

3:20 PM **Christian Ballot:** *Generalizations of a theorem of Kimberling on Beatty sequences*

3:40 PM **Break**

4:00 PM **Problem Session** by Clark Kimberling

Saturday, July 2

	Optional D-day and coastal tour
8:00 AM	Bus pickup at place Saint-Pierre
8:15 AM	Bus pickup at Résidence Côte de Nacre
11:00 AM	Guided visit of Omaha Beach Cemetery
3:30 PM	Circular Movie Theater
$\sim$ 6:00 PM	Return to Caen

Monday, June 27

Notes & Extensions for a Remarkable Continued Function

Peter G. Anderson

Let the Fibonacci words be $w_1 = 0, \quad w_2 = 1, \quad w_{n+1} = w_n w_{n-1}$. Consider them as integers expressed in binary. It is known that for $n \geq 2$ the numbers $0.\overline{w_n} = \frac{w_n}{2^{F_n}-1}$ have the continued fraction $[0; 2^0, 2^1, 2^1, 2^2, 2^3, 2^5, \ldots, 2^{F_{n-2}}]$. We provide a framework for proving and generalizing this using Fibonacci-type recurrences of compositions of linear functions. We apply this to recurrences such as Pell and Tribonacci.

On Identities of Ruggles, Horadam, and Howard

Curtis Cooper

Let w_0, w_1, a and $b \neq 0$ be integers. Define

$$w_n = aw_{n-1} + bw_{n-2} \text{ for } n \geq 2.$$

In addition, define $x_0 = 2$, $x_1 = a$, and for $n \geq 2$,

$$x_n = ax_{n-1} + bx_{n-2}.$$

Horadam proved that for integers $n \geq 0$ and $k \geq 1$,

$$w_{n+2k} = x_k w_{n+k} - (-b)^k w_n.$$

This is a generalization of Ruggles' identity.

Howard generalized this result to third order recurrence relations. We will present an alternate form of Howard's identity and prove the alternate form using a different proof technique. Finally, we will generalize this result to fourth order recurrence relations.

An Exploration of Sequence A000975

Paul K. Stockmeyer

Sequence A000975 in the On-Line Encyclopedia of Integer Sequences (the OEIS) starts out 1, 2, 5, 10, 21, 42, 85, ... Its most famous appearance is perhaps as the distance between a string of n zeros and a string of n ones in the standard binary Gray code. Equivalently it is the number of moves needed to solve the Chinese Rings puzzle if the rings are moved one at a time. In this talk we examine these and other instances where this sequence arises, and confirm two instances of this sequence that had been conjectured in the OEIS.

Congruences for Bernoulli-Lucas sums

Paul Young

We give strong congruences for sums of the form $\sum_{n=0}^{N} B_n V_{n+1}$ where B_n denotes the Bernoulli number and V_n denotes a Lucas sequence of the second kind. These congruences, and several variations, are deduced from the reflection formula for p-adic multiple zeta functions.

Arithmetic properties of the convolutions of the Prouhet-Thue-Morse sequence

*Maciej Gawron***

For an integer n we denote by $s_2(n)$ the sum of digit in binary expansion of n. Let m be a positive integer. In this talk we will investigate the sequence $(t_m(n))_{n\in\mathbb{N}}$ defined by the formula

$$t_m(n) = \sum_{i_1+i_2+\ldots+i_m=n} (-1)^{\sum_{k=1}^{m} s_2(i_k)}.$$

The $(t_m(n))_{n\in\mathbb{N}}$ is a convolution of m copies of the Prouhet-Thue-Morse sequence.

Among many other results we will establish some theorems concerning asymptotic behaviour and p-adic valuations of the sequences $(t_m(n))_{n\in\mathbb{N}}$. As a main result we use a generalization of Calkin-Wilf tree to prove that every nonzero integer occurs infinitely many times in the sequence $(t_2(n))_{n\in\mathbb{N}}$. We also present some theorems about the set of values of the sequence $(t_m(n))_{n\in\mathbb{N}}$ for $m > 2$. This is joint work with P. Miska and M. Ulas.

Arithmetic properties of convolutions of binary partition function

*Piotr Miska***

The binary partition function maps every nonnegative integer n to the number $b(n)$ of representations of n as a sum of powers of 2. It was introduced by Euler and studied by Churchhouse, Rødseth, Gupta and others. The generating function of binary partition numbers is a formal inverse of generating function of well-known Prouhet-Thue-Morse sequence.

The subject of the talk will be sequences $(b_m(n))_{n\in\mathbb{N}}$ given by the formula

$$b_m(n) = \sum_{i_1+\ldots+i_m=n} b(i_1)\cdot\ldots\cdot b(i_m)$$

for $m \in \mathbb{N}_+$. The number $b_m(n)$ has a natural combinatorial interpretation. Indeed, $b_m(n)$ counts the number of representations of the integer n as the sum of powers of 2 and that each summand can have one of the m-colors.

During the talk we will present some arithmetic properties of numbers $b_m(n)$. We will start with recurrence relations for them. Next we will show 2-adic valuations of numbers $b_{2^k-1}(n)$, where $k \in \mathbb{N}_+$. Using identities and congruences concerning generating functions of sequences $(b_m(n))_{n\in\mathbb{N}}$ we will establish congruences involving numbers $b_m(n)$.

The talk will be based on a joint work with Maciej Gawron and Maciej Ulas.

On Gaussian Fibonacci and Gaussian Lucas numbers

Mustafa Asci

In this paper the Gaussian k Fibonacci and Gaussian k Lucas Numbers are defined and studied. Also the generating functions, the Binet formulas, the summation formulas, matrix and determinant representations of these numbers are given. Finally some relations between Gaussian k Fibonacci and Gaussian k Lucas numbers are proved.

Reciprocal sums of sequences involving balancing and Lucas-balancing numbers

Takao Komatsu

Many authors studied bounds for reciprocal sums involving terms from Fibonacci and other related sequences. The purpose of this paper is to provide bounds for reciprocal sums with terms from balancing and Lucas-balancing numbers.

The Period Modulo Product of Consecutive Balancing Numbers

Bijan Patel

A natural number n is a balancing number with balancer r if they are the solutions of the Diophantine equation $1 + 2 + \cdots + (n-1) = (n+1) + (n+2) + \cdots + (n+r)$. Balancing numbers satisfy the linear recurrence $B_{n+1} = 6B_n - B_{n-1}$ and the non-linear recurrence $B_n^2 - B_{n+1}B_{n-1} = 1$, where B_n denotes n^{th} balancing number.

Periodicity is an important aspect of number sequences. Many authors have studied the periodicity of different number sequences such as Fibonacci sequence, Lucas sequence, balancing sequence etc. Panda et al. studied the periodicity of balancing numbers modulo primes and examined the periodicity modulo terms of certain sequences which exhibits important results, some of them are identical with corresponding results of Fibonacci numbers while some others are more interesting. The period modulo m, denotes by $\pi(m)$, is the smallest positive integer t for which $(B_t, B_{t+1}) \equiv (0, 1)\ (mod\ m)$ whereas the rank $\alpha(n)$ of a natural number n is defined as the smallest natural number k such that n divides B_k. For instance, $\pi(B_n) = 2n$ and $\alpha(B_n) = n$ for all $n > 1$.

Some interesting relations between the period, rank and order of the balancing numbers modulo a positive integer are recently established by Patel et al.

One such important relation is that the period of the sequence of balancing numbers is equal to the product of the rank of apparition and its order. In a subsequent paper, Ray et al. have investigated the moduli for which all the residues appear with equal frequency with a single period in the sequence of balancing numbers. In this article, we study the period modulo product of consecutive balancing numbers. For instance,

$$\pi(B_nB_{n+1}B_{n+2}) = \begin{cases} 2n(n+1)(n+2), & \text{if } n \equiv 1 \pmod 2 \\ n(n+1)(n+2), & \text{if } n \equiv 2 \pmod 6 \\ 3n(n+1)(n+2), & \text{if } n \equiv (0,4) \pmod 6. \end{cases}$$

Sequences associated with balancing-like sequences

G.K. Panda and S.S. Pradhan

A natural number n is called a balancing number if it satisfies the Diophantine equation $1+2+\cdots+(n-1) = (n+1)+\cdots+(n+r)$ for some natural number r, called the balancer corresponding to n. If n is a balancing number then $8n^2+1$ is a perfect square and $\sqrt{8n^2+1}$ is called a Lucas-balancing number. The sequences of balancing and Lucas-balancing numbers are solutions of the binary recurrence $x_{n+1} = 6x_n - x_{n-1}$ with initializations $x_0 = 0, x_1 = 1$ and $x_0 = 1, x_1 = 3$ respectively. All balancing numbers except 1 are composite and are products of Pell and associated Pell numbers of same order. Further, every balancer is a cobalancing number and each cobalancing number n is a solution of the Diophantine equation $1+2+\cdots+n = (n+1)+\cdots+(n+r)$. The sequence of cobalancing numbers are solutions of the binary recurrence $x_(n+1) = 6x_n - x_{n-1} + 2$ with initializations $x_0 = x_1 = 0$. For each cobalancing number n, $\sqrt{8n^2+8n+1}$ is a perfect square and $\sqrt{8n^2+8n+1}$is called a Lucas-cobalancing number. The balancing-like sequences defined as $x_{n+1} = Ax_n - x_{n-1}$ with initializations $x_0 = 0, x_1 = 1$ (where $A > 2$ is a natural number) are natural generalizations of the balancing sequence. It is an interesting idea to construct Lucas-balancing-like, cobalancing-like and Lucas-cobalancing-like sequences from balancing-like sequences and to see whether these sequences behave like Lucas-balancing, cobalancing and

Lucas-cobalancing sequences respectively. Further, from each balancing-like sequence, it will be interesting extract two sequences (comparable to Pell and associated Pell sequences) such that the product of these sequences is equal to the corresponding balancing-like sequence.

Infinite sums of weighted Fibonacci numbers of order k

Spyros Dafnis and Andreas Philippou

For integers $m \geq 0$ and $k \geq 2$, set $\alpha_{m,k} := \sum_{n=1}^{\infty} \frac{n^m F_n^{(k)}}{2^{n+k-1}}$, where $F_n^{(k)}$ is the Fibonacci sequence of order k or k-generalized Fibonacci sequence. It is shown that $\alpha_{0,k} = 1$, $\alpha_{1,k} = 2^{k+1}-k-1$, $\alpha_{2,k} = 2^{k+1}(2^{k+2}-4k-3)+k^2+2k-1$, and $\alpha_{m,k} = 1 + \sum_{r=0}^{m-1} \binom{m}{r} \sum_{i=1}^{k} 2^{k-i} i^{m-r} \alpha_{r,k}$, which generalize recent results on weighted Fibonacci sums by Benjamin, Neer, Otero, and Sellers.

Convolution identities for Fubini polynomials

Levent Kargin

In this talk, we define two variable Fubini polynomials and give several convolution and symmetric identities for these polynomials by means of the generating function technique. We also study some special cases involving Apostol-Bernoulli and Apostol-Euler polynomials.

[1] T.M. Apostol, On the Lerch zeta function, Pacific J. Math., 1 (1951) 161–167.

[2] K. N. Boyadzhiev, A series transformation formula and related polynomials, Int. J. Math. Math. Sci., 23 (2005) 3849-3866.

[3] A. Dil and V. Kurt, Investigating geometric and exponential polynomials with Euler-Seidel matrices, J. Integer Seq. 14-4 (2011), Article 11.4.6.

[4] Q.M. Luo, Apostol–Euler polynomials of higher order and Gaussian hypergeometric functions, Taiwanese J. Math. 10 (2006) 917–925.

[5] S. M. Tanny, On some numbers related to the Bell numbers, Canad.Math. Bull., 17 (1974) 733-738.

Explicit Formulas for Weighted Sums of Squares via Generalized Eulerian Polynomials

Claudio Pita Ruiz

By using a generalization of Eulerian polynomials, together with the Z-transform, we obtain explicit expressions for some weighted sums of squares of integers. It turns out that these formulas are related, in a natural way, with Primitive Pythagorean Triples[1].

Convolution Identities for Degenerate Bernoulli Polynomials

Mehmet Cenkci

We establish some convolution identities for degenerate Bernoulli polynomials similar to those satisfied by ordinary Bernoulli polynomials.

[1] These results are also obtained, working with a different approach, in the paper C. Pita, *Explicit Formulas for Some Rational Weighted Sums of Powers of Integers*, Journal of Combinatorics and Number Theory, **7** (1), 13–55.

Combinatorial Identities via Matrix Factorization

Marc Chamberland

The LU factorization is an effective tool in numerical linear algebra. By performing matrix factorizations symbolically, however, beautiful identities can be constructed. This talk will showcase this approach and the formulas produced connected to the Fibonacci numbers, binomial coefficients, q-series, Legendre polynomials, and more.

Tuesday, June 28

Some Surprising Lacunary Binomial Sums

William Webb

We all know that if you add all of the elements in the nth row of Pascals triangle, the sum is simply 2^n. What happens if we skip terms and include only every jth term? In a 1969 paper in the Fibonacci Quarterly, George Andrews gave a rather surprising formula for a sum involving every 5th term. Here, we give formulas for some other such lacunary sums, and address the question of how surprised we should be by the answers.

On the rate of p-adic convergence of alternating sums of powers of binomial coefficients

Tamas Lengyel

Let $m \geq 1$ be an integer and p be an odd prime. We study alternating sums and lacunary sums of mth powers of binomial coefficients from the point of view of arithmetic properties. We develop new congruences and prove the p-adic convergence of some subsequences and that in every step we gain at least three more p-adic digits of the limit. These gains are exact under some explicitly given condition. The main tools are congruential and divisibility properties of the binomial coefficients.

Balanced binary triangles generated from periodic sequences

*Jonathan Chappelon***

A *binary Steinhaus triangle* of size n is a down-pointing triangle, with n rows, whose first row is composed of n elements of $\mathbb{Z}/2\mathbb{Z}$ and built with the same local rule as the standard Pascal triangle modulo 2: each entry of the triangle (not on the first row) is the sum modulo 2 of the two directly above it. In 1963 [2], Hugo Steinhaus posed the problem to determine whether there exist, for all positive integers n such that $n \equiv 0$ or $3 \bmod 4$ (when the triangular number $\binom{n+1}{2}$ is even), a binary Steinhaus triangle of size n containing as many 0's and 1's. In 1972 [1], Heiko Harborth gave a positive solution to this problem. He proposed an explicit construction of such binary triangles of size n for all integers n such that $n \equiv 0$ or $3 \bmod 4$. The method introduced by Harborth is based on the search for pseudo-periodic sequences generating triangles where the sequence of rows is also pseudo-periodic.

Here, we do not only consider the case when the total number of elements in the triangle is even. A binary triangle is said to *balanced* if the absolute value of the difference between the number of 0's and 1's in it is at most 1. In this talk, we propose a refinement of the Harborth method that permits us to show the existence of purely periodic sequences that generate balanced binary Steinhaus triangles of size n, and this for all positive integers n.

This is not the only result that this method gives. We can also consider the *generalized Pascal triangles*, that are up-pointing triangles defined with the same local rule, or triangles defined with other local rules such as *Stirling triangles* of the first kind and of the second kind. This method can be adapted for proving the existence of balanced such triangles for all sizes.

[1] H. Harborth. Solution of Steinhauss problem with plus and minus signs. *J. Combin. Theory Ser. A*, 12 : 253–259, 1972.

[2] H. Steinhaus. One hundred problems in elementary mathematics. *Basic Books Inc. Publishers, New York*, 1964, pp. 47–48.

On Candido Identity

Zvonko Cerin

Candido discovered that twice the sum of fourth powers of three consecutive Fibonacci numbers is the square of the sum of their squares. We improve this in two directions. Instead of Fibonacci we use Horadam numbers and replace indices n, $n+1$, $n+2$ with n, $n+k$, $n+2k$, where n and k are any integers.

Mod p equivalence classes of linear recurrences of degree two

Miho Aoki

Let p be a prime number, T be an integer and $N = 1, -1$. We introduce modulo p equivalence relations for the set of linear recurrence sequences of degree two, which are defined by arbitrary integers G_0, G_1 and $G_n = TG_{n-1} - NG_{n-2}$.

We determine the initial terms G_0, G_1 of all the representatives of the equivalence classes in which any term is not divisible by p, and give the number of the equivalence classes. Furthermore, we explain the relation between our modulo p equivalence classes and without modulo p equivalence classes given by R. R. Laxton.

A new approach to the Golay-Rudin-Shapiro sequence and variations

Pantelimon Stănică

In this talk we give a Boolean functions approach to the classical Golay-Rudin-Shapiro sequence r_n defined by counting the number of occurrences of the pattern 11 in the binary expansion of n. Using the same method, we also look at other Golay-Rudin-Shapiro-like sequences defined by counting occurrences of 00, 01, 10 in the binary expansion of n. We regard a truncation of such sequences as part of the truth table of a Boolean function. For each such function, we find the algebraic normal form and its weight, consequently, showing that some of them are, in fact, perfect nonlinear, that is, bent (and in odd dimension, semibent) Boolean functions, that is, they have flat (or almost flat) Walsh-Hadamard spectrum. As a consequence, we find partial sums of the classical Golay-Rudin-Shapiro sequence for upper bounds of weight ≤ 3. Further, we show that using these new sequences we can generate Golay complementary pairs, which was the motivation for the original 1950 Golay-Rudin-Shapiro sequence.

On Markoff numbers

Anitha Srinivasan

The Markoff conjecture states that given a positive integer c, there is at most one triple (a, b, c) of positive integers with $a \leq b \leq c$ that satisfies the equation $a^2 + b^2 + c^2 = 3abc$. The conjecture is still unproven but known to be true in some special cases (such as when c is a prime power). We present some conjectures in quadratic fields that are equivalent to this conjecture. We also highlight connections between Markoff numbers and (generalized) Lucas sequences, suggesting a possibly new approach to this century old conjecture.

The Combinatorial Nature of Numerical Representation

Nathan Hamlin

Some clarications on the combinatorial nature of representation are here presented. This is related to the foundations of digital representations of integers generally, and is thus also of interest in clarifying what numbers are and how they are used in pure and applied mathematics. The author hopes this work might help the nature of the Generalized Knapsack Code be better understood by mathematicians and computer scientists.

What I tell you K times is True ...

Paul Cull

Some years ago while investigating generalized Zeckendorf representations, i.e. representations of integers as binary sums of kth order Fibonacci numbers, we found that the fraction of 0's in the representation was monotone increasing function of the number of bits used. It was relatively easy to show that this behavior was to be expected asymptotically, but was there an easy way to show that this fraction was *always* increasing? Specifically, could one find a K (presumably bigger than the order k) so that if the fraction was increasing for K consecutive steps, then it would always be increasing? Here, we introduce the **SP** (sorta positive) polynomials and show that if the characteristic polynomial for a difference equation is a factor of an **SP** polynomial of degree K then if the ratios of any two solutions to the equation are K in row increasing then the ratios are always increasing. For the kth order Fibonacci sequences $K = k + 1$. For our Zeckendorf problem, the characteristic polynomial is the square of a Fibonacci characteristic polynomial (i.e. degree $= 2k$) and we show that $K = 2k + 2$.

Convergence Rates in Generalized Zeckendorf Decompositions

*Steven J. Miller, Zhao Pan**, Huanzhong Xu***

We report on two projects on generalized Zeckendorf decompositions. Zeckendorf proved any integer can be decomposed uniquely as a sum of non-adjacent Fibonacci numbers, F_n. For Fibonacci numbers, the fraction of gaps that are size k, P_k $(k \geq 2)$, over all $m \in [F_n, F_{n+1})$ approaches $1/\phi^k$ as $n \to \infty$. Recently Bower et al. provided an explicit computation of P_k for all positive linear recurrence sequences $\{G_n\}_{n\in\mathbb{N}}$ from the recurrence relation. We extend these results and prove that, if $P_k(n)$ denotes the fraction of gaps that are size k for $m \in [G_n, G_{n+1})$, then $|1 - \frac{P_k(n)}{P_k}| = \frac{k+O(1)}{n+O(1)}$. Furthermore, we explicitly compute the $O(1)$ terms, and show that for a fixed g, for almost all $m \in [G_n, G_{n+1})$ the number of gaps of length g converges to a Gaussian as $n \to \infty$. This is joint work with Ray Li.

The second project concerns positive linear recurrence sequence (PLRS), which are of the form $H_{n+1} = c_1 H_n + \cdots + c_L H_{n+1-L}$ with each $c_i \geq 0$ and $c_1 c_L > 0$, with appropriately chosen initial conditions. There is a notion of a legal decomposition generalizing the non-adjacency condition from the Fibonacci numbers (roughly, given a sum of terms in the sequence we cannot use the recurrence relation to reduce it) such that every positive integer has a unique legal decomposition of terms in the sequence. Previous work proved not only that a decomposition exists, but that the number of summands $K_n(m)$ in legal decompositions of $m \in [H_n, H_{n+1})$ converges to a Gaussian. Using partial fractions and generating functions it is easy to show the mean and variance grow linearly in n: $an + b$ and $cn + d$; the difficulty is proving a and c are positive. Currently the only way to do this requires delicate analysis of polynomials related to the generating functions and characteristic polynomials, and is algebraically cumbersome. We introduce new, elementary techniques that bypass these issues. The key insight is to use induction and bootstrap bounds through conditional probability expansions to show the variance is unbounded, and hence $c > 0$ (the mean is handled easily through a simple counting argument). These arguments can be generalized to other sequences that have a notion of legal decomposition and

unique decomposition. (Note: Zhao Pan, Huanzhong Xu are eligible for the Bruckman prize.)

De Hanoï à Londres, aller et retour

Andreas Hinz

La Tour de Hanoï est un jeu scientifique inventé vers le fin du 19ième siècle par l'arithméticien français, Édouard Lucas, qui est aussi connu pour avoir nommé la suite de Fibonacci. Très vite le casse-tête fut employé par les psychologues afin de tester les capacités cognitives des individus ; ils introduisirent aussi une variante, appelée la Tour de Londres. Ces tests seront présentés avec quelques observations sur leurs fondements mathématiques dans la théorie des graphes.

Numbers You Can Eat: nature's numbers that you see and eat in your everyday fruit and vegetable

Ron Knott

Ron's presentation will include some surprises.

Mathemagics

Arthur Benjamin

Dr. Arthur Benjamin is a mathematician and a magician. In his entertaining and fast-paced performance, he will demonstrate and explain how to mentally add and multiply numbers faster than a calculator, how to figure out the day of the week of any date in history, and other amazing feats of mind. He has presented his mixture of math and magic to audiences all over the world.

Wednesday, June 29

Variations on the Fibonacci binary sequence

Jean-Paul Allouche

The Fibonacci binary sequence is defined as follows. First consider the substitution rule where 0 is replaced by 01, and 1 is replaced by 0. Applying this substitution rule to a finite string of symbols 0 and 1 means applying it "in parallel" to each 0 and each 1 of the string. For example, applying it to 001 gives 01010 (the first two zeroes where replaced by 01 while simultaneously the last 1 was replaced by 0). Then start from 0 and apply iteratively the substitution rule; you obtain successively

0

01

010

01001

01001010

...

Note that the length of the strings you obtain is $1, 2, 3, 5, 8, \ldots$ where you might recognize one of your favorite sequences, and that the sequence of strings "converges" in a reasonable sense to an infinite sequence

$$\mathbf{F} = 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ \ldots$$

which is thus called the Fibonacci binary sequence.

We will see how this sequence is in some sense one of the "simplest" non-periodic sequences, how it can be obtained by playing billiard on a square and what other sequences share these two properties. Finally we will indicate unexpected inequalities involving these sequences and their shifts; for example if we let S denote the shift on sequences, so that $S\mathbf{F} = 1\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ \ldots$, $S^2\mathbf{F} = 0\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ \ldots$, and if $\leq$ is the lexicographical order on binary sequences, then for all $k \geq 1$

$$0\ \mathbf{F} \leq S^k(\mathbf{F}) \leq 1\ \mathbf{F}.$$

Combinatorial and divisibility properties of generalized Lah numbers and their sums

Gabriella Rácz[**]

Stirling numbers of the second kind are of basic importance in enumerative combinatorics. They have various generalizations and variants. Among them, r-Stirling numbers were independently defined by A. Z. Broder and R. Merris, when r distinguished elements have to be in distinct blocks. Lah numbers, sometimes called Stirling numbers of the third kind, count the number of partitions into a fixed number of ordered sets. Analogously to r-Stirling numbers, r-Lah numbers also can be defined. We do this in the first part of our talk.

Another fundamental objects in enumerative combinatorics are Bell numbers which count the partitions of a finite set of given size, in other words, they are the sums of Stirling numbers of the second kind with a fixed upper parameter. István Mező defined the r-generalization of the Bell numbers, the so called r-Bell numbers. As the sum of the r-Lah numbers, the summed r-Lah numbers can be introduced similarly. We do it in detail in the second part of our talk.

We also study some combinatorial and divisibility properties of the r-Lah numbers and the summed r-Lah numbers.

This is a joint work with Gábor Nyul.

Recent results on generalizations of Bell numbers

*Gábor Nyul***

Bell numbers play a central role in enumerative combinatorics. After an overview of fundamentals of Bell numbers, we discuss our recent results on certain generalizations. For instance, we study r-Bell numbers, associated Bell numbers, Bell numbers for graphs, Dowling numbers and their combinations. We present their properties with a special emphasis on the connections of these variants.

These results are joint with Eszter Gyimesi.

A Bijective Proof of a Derangement Recurrence

*Arthur T. Benjamin and Joel Ornstein***

The number of permutations of order n with no fixed points is called the nth derangement number, and is denoted by D_n. It is well-known that for $n > 1$, the derangement numbers satisfy the recurrence $D_n = nD_{n-1} + (-1)^n$, but as Richard Stanley notes, "considerably more work is needed to prove it combinatorially." We present what we believe to be the simplest combinatorial proof of this recurrence.

Thursday, June 30

Two applications of the bijection on Fibonacci set partitions

Augustine O. Munagi

Fibonacci partitions refer to the partitions of $\{1, 2, \ldots, n\}$ into blocks of nonconsecutive elements. The name was coined by Prodinger because there are as many nonconsecutive subsets of $\{1, 2, \ldots, n\}$ as the Fibonacci number F_{n+1} [Fibonacci Quart. **19** (1981), 463–465]. In this talk we discuss an application of the bijection between Fibonacci partitions and standard partitions to a new formula for the number of partitions with no circular successions, that is, pairs of elements $a < b$ in a block satisfying $b - a \equiv 1 \pmod{n}$. Then we demonstrate an application of an extended form of the bijection.

Tilings, Continued Fractions, Derangements, Scramblings, and e

Barry Balof and Helen Jenne

A classic, but perhaps less well-known formula of Euler gives the following continued fraction representation

$$e = 2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{2}{3 + \cfrac{3}{4 + \cfrac{4}{5 + \ddots}}}}}$$

In their book *Proofs That Really Count*, Benjamin and Quinn ask about the combinatorial implications of this formula. By looking at the sequences of convergents, we are able to interpret this formula in terms of derangements and scramblings (permutations which leave no fixed adjacencies).

Combinatorial interpretation of binomial coefficient analogues related to 3-Fibonacci sequence

Imène Benrabia

Inspiring from binomials $\binom{n}{k}$, Fibonomial coefficients $\binom{n}{k}_F$ are built up from Fibonacci factorials in the same way that binomial coefficients are formed from standard factorials. Using the 3-Fibonacci sequence, ($\phi_0 = \phi_1 = \phi_2 = 1$ and for all $n \geq 3$, $\phi_n = \phi_{n-1} + \phi_{n-3}$), we suggest a recurrence relation of the 3-Fibonomial coefficients.

The classical Fibonomial coefficients associated to Fibonacci sequence satisfies, for $n \geq 2$, the following recurrence

$$\binom{n}{k}_F = F_{n-k+1}\binom{n-1}{k-1}_F + F_{k-1}\binom{n-1}{k}_F.$$

From that recurrence, Sagan and Savage gave a combinatorial interpretation of the Fibonomial coefficients, basing on the fact that the binomial coefficient $\binom{n}{k}$ counts lattice paths from $(0,0)$ to $(k, n-k)$, with exactly n steps (k horizontal steps and $n-k$ vertical ones), and that the Fibonacci number F_{n+1} counts the number of ways to tile a board of length n.

Our aim is to introduce the 3-Fibonacci sequence, and give a recurrence relation for the 3-Fibonomial coefficients, and a combinatorial interpretation using tiling approach by linear triominos and squares.

[1] A. T. Benjamin, & E. Reiland. (2014).Combinatorial proofs of fibonomial identities,*Fibonacci Quarterly*.

[2] B. E. Sagan & C. D. Savage. (2009). Combinatorial interpretation of binomial coefficient analogues related to Lucas sequences, *arXiv:0911.3159v1*

[3] A. T. Benjamin & J. J. Quinn. (2003). Proofs That Really Count: The art of Combinatorial proofs, *Washington DC: Mathematical Association of America.*

Two Combinatorial Interpretations for the Fibonomial

Dale Gerdemann

The fibonomial $\binom{n}{k}_F$ is defined like the binomial, but with the factorials replaced by Fibonacci factorials.

$$\binom{n}{k}_F = \frac{F_n F_{n-1} \cdots F_1}{F_{n-k} F_{n-k-1} \cdots F_1 F_k F_{k-1} \cdots F_1} = \frac{n!_F}{(n-k)!_F k!_F} \tag{1}$$

An alternative Pascal-Triangle type of formula (below) can be derived from this by a single application of $F_n = F_{k+(n-k)} = F_{k+1}F_{n-k} + F_k F_{n-k-1}$ to replace F_n in the numerator.

$$\binom{n}{k}_F = F_{n-k-1}\binom{n-1}{k-1}_F + F_{k+1}\binom{n-1}{k}_F \tag{2}$$

This recursive formula leads directly to a weighted tiling interpretation of the fibonomial. In Sagan and Savage [1], these weights are interpreted as square-domino tilings to the left or below a lattice path through a rectangle. In this talk, I present an alternative lattice-path-in triangle approach. This alternative is intuitive since the triangle corresponds directly to the fibonomial number triangle.

[1] B. Sagan and C. Savage. Combinatorial interpretations of binomial coefficient ana- logues related to lucas sequences. Integers, 10(A52):697703, 2010.

Fibonacci-Like Sequences and Chebysev Polynomials

*Murat Sahin***

Let $\{a_{i,j}\}$ be real numbers for $0 \leqslant i \leqslant r-1$ and $1 \leqslant j \leqslant 2$, and define a sequence $\{v_n\}$ with initial conditions v_0, v_1 and conditional linear recurrence relation $v_n = a_{t,1}v_{n-1} + a_{t,2}v_{n-2}$ where $n \equiv t \pmod r$ $(n \geqslant 2)$. We establish a connection between Fibonacci-like sequences $\{v_n\}$ and Chebyshev polynomials. Then, using Chebyshev polynomials, we obtain the complex factorization of the sequence $\{v_n\}$ so that we extend some recent results and solve some open problems.

On the 2*-class group of* $\mathbb{Q}(\sqrt{5pF_p})$ *where* F_p *is a prime Fibonacci number*

Mohammed Taous

Let F_p be a prime fibonacci number where $p > 5$. Put $\mathbf{k} = \mathbb{Q}(\sqrt{5pF_p})$ and let $\mathbf{k}_1^{(2)}$ be its Hilbert 2-class field. Denote by $\mathbf{k}_2^{(2)}$ the Hilbert 2-class field of $\mathbf{k}_1^{(2)}$ and by $G = \mathrm{Gal}(\mathbf{k}_2^{(2)}/\mathbf{k})$ the Galois group of $\mathbf{k}_2^{(2)}/\mathbf{k}$. In this work, we determine the structure of the 2-class group of $\mathbf{k}$ and we study the metacyclicity of G.

A New Bound for the zeros of R*-Bonacci Polynomials*

*Ö. Öztunç Kaymak*** and N. Yilmaz Özgür

In this study, we find a new bound involving well-known Fibonacci numbers concerning the location of the zeros of R-Bonacci Polynomials. Also, we

obtain an algorithm to see our results are better than the known bounds for these polynomials. (This is joint work with Yilmaz Özgür)

P-adic root separation for quadratic and cubic polynomials

Tomislav Pejković

We study p-adic root separation for quadratic and cubic polynomials with integer coefficients. The quadratic and reducible cubic polynomials are completely understood, while in the irreducible cubic case, we give a family of polynomials with the bound which is the best currently known.

Boundedness of Periodic Points of Rational Functions with Good Reduction Everywhere by Using Fibonacci Numbers

Chatchawan Panraksa

Morton and Silverman's uniform boundedness conjecture states that the number of periodic points of any rational function with rational coefficients is bounded by a constant that depends only on the degree of the function. The conjecture is still unsolved even for quadratic polynomials. However, if we consider a family of rational functions with good reduction everywhere, then the periods are uniformly bounded and do not depend on the degree of the functions. It can be proved by using a sequence related to Fibonacci numbers.

Rational polynomials that take integer values on the Fibonacci numbers

Keith Johnson

An algorithm for constructing a basis for the set of polynomials with rational coefficients which take integer values when evaluated at Fibonacci numbers will be described. Using this, a method for computing the p-adic valuative capacity of the Fibonacci will be presented.

A variant of the Brocard-Ramanujan equation for Lucas sequences

Márton Szikszai

Brocard, and independently Ramanujan, posed the equation

$$n! + 1 = m^2$$

in positive unknown integers n and m. It is often mentioned as the Brocard-Ramanujan equation. Solutions to it are called Brown numbers, the only known are $(n, m) = (4, 5)$, $(5, 11)$ and $(7, 71)$. Erdős conjectured that this list is complete. Beside various generalizations, in recent years, several variants have been considered. For instance, Dabrowski and Ulas posed the problem of characterizing all monotone increasing functions $f : \mathbb{N} \to \mathbb{N}$ such that the equation

$$f(1)f(2)\dots f(n) + 1 = f(m)^2$$

has no solution in positive unknown integers n and m. Partial answers to this problem include for example the case when $f(n)$ is the n-th Fibonacci number. In this talk, we address a similar problem. Let $u = (u_n)_{n=0}^{\infty}$ be a Lucas sequence and consider the equation

$$u_{n_1}u_{n_2}\dots u_{n_k} + 1 = u_m^2$$

in unknown non-negative integers $(k, m, n_1, n_2, \dots, n_k)$ where $k \geq 2$ and $n_1 < n_2 < \dots < n_k$. We completely solve the above equation for parametric families. Further, we also consider its analogue with associated sequences and obtain similar results. Our main tools include factorization properties, primitive divisors and growth estimates.

Perfect powers in Fibonacci and Lucas polynomials in finite fields

*Hidetaka Kitayama*** *and Daisuke Shiomi***

Let $(F_n)_{n\geq 0}$ and $(L_n)_{n\geq 0}$ be the Fibonacci and Lucas sequences defined by

$$F_0 = 0,\ F_1 = 1,\ F_{n+2} = F_{n+1} + F_n \text{ for } n \geq 0,$$
$$L_0 = 2,\ L_1 = 1,\ L_{n+2} = L_{n+1} + L_n \text{ for } n \geq 0.$$

They are the most typical linear recurrence sequences and have attracted much attention from various aspects of mathematics. In 2006, Bugeaud, Mignotte and Siksek determined all perfect powers in Fibonacci and Lucas sequences. In this talk, we give an analogue of their results for Fibonacci and Lucas polynomials over finite fields.

Let p be a prime, and let $\mathbb{F}_p[T]$ be the polynomial ring over the finite field $\mathbb{F}_p$ with p elements. The Fibonacci polynomials $(F_n(T))_{n\geq 0}$ and the Lucas polynomials $(L_n(T))_{n\geq 0}$ are defined by the following recurrence relations:

$$\begin{aligned} F_0(T) &= 0,\ F_1(T) = 1,\ F_{n+2}(T) = TF_{n+1}(T) + F_n(T) \quad \text{for } n \geq 0, \\ L_0(T) &= 2,\ L_1(T) = T,\ L_{n+2}(T) = TL_{n+1}(T) + L_n(T) \quad \text{for } n \geq 0. \end{aligned}$$

Main results of this talk are as follows:

Theorem 1 *Let $k \geq 2$ and $n \geq 1$ be integers. Then the equation $X^k = L_n(T)$ has a solution in $\mathbb{F}_p[T]$ if and only if $(k, n) = (p^r, p^r t)$ for some $r, t \in \mathbb{N}$.*

Theorem 2 *Let $k \geq 2$ and $n \geq 2$ be integers. We define $t = 2k$ if $p \neq 2$, and $t = k$ if $p = 2$.*

(1) *If* $(k,p)=1$*, then the equation* $X^k=F_n(T)$ *has a solution in* $\mathbb{F}_p[T]$ *if and only if* $n=p^{rm}$ *for some* $m\in\mathbb{N}$*, where* r *is the order of* p *in* $(\mathbb{Z}/t\mathbb{Z})^\times$*.*

(2) *If* $(k,p)>1$*, then the equation* $X^k=F_n(T)$ *has a solution in* $\mathbb{F}_p[T]$ *if and only if* $(p,k,t)=(2,2,2l+1)$ *for some* $l\in\mathbb{N}$*.*

Exact Divisibility Properties of Some Subsequences of the Mersenne Numbers

*Aram Tangboonduangjit***

We prove that the sequence of Mersenne numbers $M_n=M(n)=2^n-1$ has the property that M_n^k exactly divides $M(nM(nM(\cdots nM(n)\cdots)))$ where n appears k times in this formulation. The Mersenne-like numbers are defined and it can be shown that the similar result holds for these numbers as well. This work serves as a generalization of a previous work by the authors where the Fibonacci numbers are the central object in that study.

Diophantine triples of Fibonacci numbers

Florian Luca

A Diophantine triple is a set of three positive integers $\{a,b,c\}$ such that $ab+1$, $ac+1$, $bc+1$ are all three squares. A parametric example is $\{F_{2n},F_{2n+2},F_{2n+4}\}$ for all positive integers n. In my talk, I will show that if $n\geq 2$ and $\{F_{2n},F_{2n+2},F_k\}$ is a Diophantine triple then $k\in\{2n-2,2n+4\}$ except for the case $n=2$ when also $k=2$ is possible. The proof uses linear forms in logarithms of algebraic numbers. (coauthors: Bo He , Alain Togbé)

Shifted Euler-Seidel Matrices

Ayhan Dil

In this talk defining Shifted Euler-Seidel matrices we will generalize the Euler-Seidel matrices method. Owing to this generalization one can investigate any sequences (s_n) which have two term linear recurrences as $s_{m+n} = \alpha s_{m+n-1} + \beta s_{n-1}$ (α and β are real parameters and $n, m \in \mathbb{Z}^+$). By way of illustration, we give some examples related to the Fibonacci p-numbers.

Generalized Fibonacci Numbers with Matrix Method

Thotsaporn Thanatipanonda

Generalizing the Fibonacci numbers by matrices has been known to be very effective and given rise to a great number of new and surprising identities. In this note, another attempt on utilizing the matrices to study the Fibonacci numbers has been made once again with the purpose to unify and link the identities which seem unrelated. Certain known identities are proved by this proposed matrix method. Moreover, new identities such as the Vajda-like ones have been discovered. (This is a joint work with Chatchawan Panraksa)

k-order linear recursive sequences and the Golden Ratio

*Tamás Szakács***

In this paper, we investigate sequences $\{G_{n+1}/G_n\}_{n=1}^{\infty}$ which are approaching the Golden Ratio, where $\{G_n\}_{n=0}^{\infty}$ is a k-order linear recursive sequence. Let

$\{x_n\}_{n=0}^{\infty}$ and $\{y_n\}_{n=0}^{\infty}$ be convergent sequences of real numbers with $\lim_{n\to\infty} x_n = y_n = z$, we say that $\{y_n\}_{n=0}^{\infty}$ converges quicker than $\{x_n\}_{n=0}^{\infty}$ if

$$\lim_{n\to\infty} \frac{y_n - z}{x_n - z} = 0.$$

We prove theorems about quicker convergence, using the well known Fibonacci sequence. Our result is that $\left\{\frac{G_{n+1}}{G_n}\right\}_{n=1}^{\infty}$ converges quicker to the Golden Ratio than $\left\{\frac{F_{n+1}}{F_n}\right\}_{n=1}^{\infty}$, if all the roots of the characteristic polynomial of $\{G_n\}_{n=0}^{\infty}$ has the property that their absolute values are all less than $\frac{1-\sqrt{5}}{2}$, which is the root of the characteristic polynomial of the Fibonacci sequence.

[1] Mtys, F., *Sequence transformations and linear recurrences of higher order, Acta Mathematica et Informatica Universitatis Ostraviensis 9 (2001), 45-51.*

[2] Mtys F., *Linear recurrences and rootfinding methods, Acta Academiae Paedagogicae Agriensis, Sectio Mathematicae 28 (2001), 27-34.*

[3] F. Gatta and A. D'Amico, *Sequences $\{H_n\}$ for which H_{n+1}/H_n approaches the Golden Ratio, Fibonacci Quart. 46/47 (2008/2009), no. 4, 346-349.*

[4] T. Komatsu, *Sequences $\{H_n\}$ for which H_{n+1}/H_n approaches an irrational number, Fibonacci Quart. 48 (2010), no. 3, 265-275.*

Friday, July 1

Polynomial Trees and Subtrees

Clark Kimberling and Peter Moses

Let T^* be the set of polynomials in x generated by these rules: 0 is in T^*, and if p is in T^*, then $p+1$ is in T^* and $x*p$ is in T^*. Let $g(0) = 0$, $g(1) = 1$, $g(2) = \{2, x\}$, and so on, so that $|g(n)| = 2^{n-1}$ for $n \geq 1$. Let $T(r)$ be the subtree of T^* obtained by substituting r for x and deleting duplicates. For various choices of r, the sequence $(|g(n)|)$ satisfies a linear recurrence relation.

The Lichtenberg Sequence

Andreas M. Hinz

The discovery of two passages from 1769 by the German Georg Christoph Lichtenberg [2] and the Japanese Yoriyuki Arima [1], respectively, sheds some new light on the early history of integer sequences and mathematical induction. Both authors deal with the solution of the ancient *Chinese rings* puzzle, where metal rings are moved up and down on a very sophisticated mechanical arrangement (see [3], Chapter 1). They obtain the number of (necessary) moves to solve it in the presence of n rings. While Lichtenberg considers all moves, Arima concentrates on the down moves only of the first ring. We will present properties of the *Lichtenberg sequence* ℓ_n, defined mathematically by the recurrence $\ell_n + \ell_{n-1} = M_n$, where M_n is the *Mersenne number* $2^n - 1$, and related sequences like the *Jacobsthal sequence*, which is the sequence of differences of ℓ. And, of course, at some point Fibonacci numbers will enter the scene [4].

[1] S. Döll, A. M. Hinz, Kyū-renkan—the Arima sequence, Advanced Studies for Pure Mathematics, to appear.

[2] A. M. Hinz, The Lichtenberg Sequence, in preparation.

[3] A. M. Hinz, S. Klavžar, U. Milutinović, C. Petr, The Tower of Hanoi—Myths and Maths, Springer, Basel, 2013.

[4] E. Jacobsthal, Fibonaccische Polynome und Kreisteilungsgleichungen, Sitzungsberichte der Berliner Mathematischen Gesellschaft **17** (1918), 43–51.

Generalization of the Tagiuri-Gould Identities to m parameters with proof by Binetization

Russell Hendel

We present and prove an infinite one-dimensional generalization of the Tagiuri and Gould identities which are obtained from the infinite one-dimensional family by letting the number of parameters equal 2 and 3 respectively. Two novelties of the paper are the following. (1) The identities naturally obtained from the one-dimensional family typically have large numbers of indices that are randomly distributed among the identity summands. Nevertheless, these identities have a proof by the book, since they are obtained from the one-dimensional family by specification of the number of parameters and their values. (2) The one dimensional family of identities is proven by Binetization, a generalization of the proof method, proof by Binet form. Binetization is based on the Dresel z-transform. We believe that the new proof method and new set of identities will establish a new and fruitful trend in Fibonacci-Lucas identities.

Areas of triangles, quadrilaterals and other polygons with vertices from various sequences

Virginia Johnson

Motivated by Elementary Problem B-1172 [Edwards] formulas for the areas of triangles, quadrilaterals and other polygons having vertices with coordinates taken from various sequences of integers are obtained.

Higher Order Boustrophedon Transforms for Certain Well–Known Sequences

Charles Cook

A review of the boustrophedon transform is presented and transforms of several familiar sequences are obtained. In addition higher transforms are also investigated. Representations of the transform will be given in terms of members of the original sequence using the Euler Up-Down number coefficients.

Why do Fibonacci numbers appear in patterns of growth in nature? Clues from modeling asymmetric cell division

Bruce Boman

While many examples of Fibonacci numbers are found in phenotypic structures of plants and animals, the dynamic processes that generate these structures have not been fully elucidated. This raises the question: What biologic

rules and mathematical laws that control the growth and organization of tissues in multi-cellular organisms give rise to these patterns of Fibonacci numbers? In nature the growth and self-renewal of cell populations leads to generation of hierarchical patterns in tissues that resemble the pattern of population growth in rabbits, which is explained by the classic Fibonacci sequence. Consequently, we conjectured a similar process exists at the cellular scale that explains tissue organization. Accordingly, we created a model (cell division type) for tissue development based on the biology of cell division that builds upon the cell maturation concept posed in the Spears and Bicknell-Johnson model (rabbit mating type) for asymmetric cell division. In our model cells divide asymmetrically to generate a mature and an immature cell. The immature cell becomes a mature cell as defined by the number of cell cycles (maturation period) between its initial generation and its first division to produce a new immature cell. Mature cells continue to divide until they become wholly mature and no longer divide and subsequently die (cell lifespan). Model output on the number of cells generated over time fits specific Fibonacci p-number sequences depending on the maturation time. Output on number of immature and mature cells for different generations at specific times is also defined by a binomial equation. A computer code was created to display model output as branching tree diagrams as a function of time. These plots and tables of model output illustrate that specific geometric patterns and ratios of immature to mature cells emerge over time based on the cell maturation period. Conclusion: Simple mathematical laws involving temporal and spatial rules for cell division begin to explain how Fibonacci numbers appear in patterns of growth in nature.

Phyllotactically Perfect Angles

Burghard Herrmann

Many plants grow regularly from a shoot axis and their leaves, petals, or cells show criss-crossing spirals. Counting parallel spirals in both directions mostly results in adjacent Fibonacci numbers. This is well known to be related to the golden angle as divergence angle between successive leaves. In this talk golden angles are generalized to the notion of *phyllotactically* *perfect angles*. They are defined by an initial angular positioning of leaves that divides the full circle into smaller and larger sectors of golden ratio. The main theorem states that each forthcoming leaf divides the widest gap between previous leaves by golden ratio. This is applied to calculate which criss-crossing spirals become visible depending on height and radial growth.

Phi and its Natural Logarithm

Christopher Brown

This talk will explore relationships between Φ (the Golden Ratio), and the natural logarithm of Φ (g). Earlier this year some of these results have been published on Dr Ron Knott's website in section 10 of the formulae pages. It will be shown how summations can be derived for g. These will be in terms of:

(i) Fibonacci Numbers and Φ

(ii) Lucas Numbers and Φ

(iii) Φ itself

Finally it will be demonstrated that g can be written as a summation which uses Fibonacci Numbers alone.

Construction of a Quasicrystalline Fivefold Structure

Frédéric Mansuy

Since their discovery in 1982 by Dan Shechtman, quasicrystals are still somewhat mysterious. The atoms they are made of arrange themselves on long distances in a fivefold symmetry that is forbidden by the rules of crystallography. The Penrose tilings are one of the rare examples presenting such a symmetry. It is therefore natural to assume that atoms in quasicrystals should be organized in a similar way. However, the only rules of placement of the basic bricks composing theses tilings are not sufficient to solve the mystery since after a certain extension, they offer several placement possibilities of these bricks thus provoking a decoherence of the system. As a consequence, there is a need for a global, fundamental, geometric logic to explain the formation of quasicrystals; this is precisely what this article proposes through a method of construction by development based on the intrinsic particularities of the infinite Fibonacci word and needing no mathematical calculation.

Properties of Fibbinary numbers with applications in Stern polynomials

Larry Ericksen

Fibbinary number sequences will be shown to have application to generalized Stern polynomials. Hyperbinary properties of their representations will be discussed and their occurrence in continued fractions will be illustrated.

Basic linear recursive matrices

Anthony G. Shannon

The essential idea in this paper it to generalize and synthesize some of the pioneering ideas of Bernstein, Lucas and Horadam on generalizations of basic and primordial Fibonacci numbers in relation to continued fractions with matrices as the unifying elements.

Convergence results for the iterated means of the denominators of Lüroth expansions

Rita Giuliano

The Lüroth expansion of $x \in (0,1]$ is

$$x = \frac{1}{d_1} + \frac{1}{(s_1)d_2} + \frac{1}{(s_1 s_2)d_3} + \cdots + \frac{1}{(s_1 \cdot \cdots \cdot s_n)d_{n+1}} + \cdots = \sum_{k=1}^{\infty} \frac{1}{\left(\prod_{h=1}^{k-1} s_h\right) d_k}.$$

where $s_n = d_n(d_n - 1)$, $n \geqslant 1$.

Let $(\Omega, \mathcal{A}, P) = \big([0,1], \mathcal{B}([0,1]), \mathbb{L}\big)$, where $\mathcal{B}([0,1])$ is the σ-algebra of Borel subsets of $[0,1]$ and $\mathbb{L}$ = Lebesgue measure in $[0,1]$. The digits $d_n(x)$ can be viewed as random variables, and as usual we shall denote them with capital letters D_n.

Denote

$$D_h^{(0)} = D_h; \qquad D_h^{(r+1)} = \frac{\sum_{k=1}^{n} D_k^{(r)}}{h}.$$

The following results are well known

Theorem 3 *As* $n \to \infty$

$$\frac{D_n^{(1)}}{\log n} \xrightarrow{P} 1.$$

Theorem 4 *As* $n \to \infty$

$$D_n^{(1)} - \log n - 1 \xrightarrow{\mathcal{L}} \mu,$$

where μ is the probability law determined by the characteristic function

$$\psi(t) = \exp\Big(- \frac{1}{2}\pi |t| - it \log |t| \Big).$$

We prove the analogues of these Theorems for the sequence of $r-$th iterated means $\left(D_h^{(r)}\right)_{h\in\mathbb{N}^*}$. The proof is via two general Theorems which can be of interest on their own.

Generalizations of a theorem of Kimberling on Beatty sequences

Christian Ballot

Here is the 2008 theorem of Clark Kimberling. Let $a(n) = \lfloor n\alpha \rfloor$ and $b(n) = \lfloor n\alpha^2 \rfloor$, where $\alpha = \frac{1+\sqrt{5}}{2}$. Each function f, composed of several a's and b's, can be expressed in the form $c_1 a + c_2 b - c_3$, where c_1 and c_2 are consecutive Fibonacci numbers determined by the number of a's and b's composing f and c_3 is a nonnegative constant.

List of participants

This is the list as of Wednesday, June 22nd. Names followed by a question mark may, or may not be able to join. Those followed by a "(no)" have very lately, for health or other reasons, declared they would not be able to join the conference.

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